

$$\int_{-\infty}^{\infty} x(at) dt = ?$$

For $a > 0$, let $u = at \Rightarrow t = \frac{1}{a}u$
 $dt = \frac{1}{a} du$

limits $t \rightarrow \infty \Rightarrow u \rightarrow \infty$
 $t \rightarrow -\infty \Rightarrow u \rightarrow -\infty$

$$\int_{-\infty}^{\infty} x(u) \frac{1}{a} du = \frac{1}{a} \int_{-\infty}^{\infty} x(u) du$$

For $a < 0$, let $u = at \Rightarrow t = \frac{1}{a}u$
 $dt = \frac{1}{a} du$

limits $t \rightarrow \infty \Rightarrow u \rightarrow -\infty$
 $t \rightarrow -\infty \Rightarrow u \rightarrow \infty$

$$\int_{\infty}^{-\infty} x(u) \frac{1}{a} du = -\frac{1}{a} \int_{-\infty}^{\infty} x(u) du$$

Hence, $\int_{-\infty}^{\infty} x(at) dt = \frac{1}{|a|} \int_{-\infty}^{\infty} x(u) du$
 for $a \neq 0$

$$u(t) \Big|_{t=nT_s} = u(nT_s) = \frac{1}{T_s} u[n]$$